Introduction to Applied Scientific Computing using MATLAB

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In this lecture, slides from Mathworks, MIT, Waterloo and Rutgers Universities are used

Arrays and Matrices

arrays and matrices are the most important data objects in MATLAB

We discuss briefly:

a) row and column vectors
b) transposition operator, '
c) colon operator, **:**
d) equally-spaced elements, linspace
e) accessing array elements
f) dynamic allocation & de-allocation
g) pre-allocation

The key to efficient MATLAB programming can be summarized in three words:

vectorize, vectorize, vectorize

and avoid all loops

Compare the two alternative computations:

```
x = [2,-3,4,1,5,8];
y = zeros(size(x));
for n = 1:length(x)
   y(n) = x(n)^2;
end
```

```
x = [2,-3,4,1,5,8];
y = x.^2;
```

element-wise exponentiation .^

ordinary exponentiation ^

answer: y = [4,9,16,1,25,64]

```
>> x = [0 1 2 3 4 5]                % row vector
x =
     0     1     2     3     4     5

>> x = 0:5                          % row vector
x =
     0     1     2     3     4     5

>> x = [0 1 2 3 4 5]'               % column vector, (0:5)'
x =
     0
     1
     2
     3
     4
     5
```

the prime operator, ', or transpose, turns row vectors into column vectors, and vice versa

caveat: ' is actually conjugate transpose,
use dot-prime, .' , for transpose w/o conjugation

```
>> z = [i; 1+2i; 1-i]      % column vector
z =
        0 + 1.0000i
   1.0000 + 2.0000i
   1.0000 - 1.0000i

>> z.'              % transpose without conjugation
ans =
        0 + 1.0000i   1.0000 + 2.0000i   1.0000 - 1.0000i

>> z'               % transpose with conjugation
ans =
        0 - 1.0000i   1.0000 - 2.0000i   1.0000 + 1.0000i

>> (z.')'           % same as (z').' , or, conj(z)
ans =
        0 - 1.0000i
   1.0000 - 2.0000i
   1.0000 + 1.0000i
```

about linspace:

```
x = linspace(a,b,N+1);
```

is equivalent to:

```
x = a : (b-a)/N : b;
```

i.e., N+1 equally-spaced points in the interval [a,b]
or, dividing [a,b] into N equal sub-intervals

$$x(n) = a + \left(\frac{b-a}{N}\right)(n-1), \quad n = 1, 2, \ldots, N+1$$

step increment

```
>> x = 0 : 0.2 : 1              % in general, x = a:s:b
>> x = linspace(0,1,6)          % see also logspace
x =
      0   0.2000   0.4000   0.6000   0.8000   1.0000
```

6 points, 5 subintervals

```
>> x = 0 : 0.3 : 1
x =
    0    0.3    0.6    0.9

>> x = 0 : 0.4 : 1
x =
    0    0.4    0.8

>> x = 0 : 0.7 : 1
x =
    0    0.7

% before rounding, (b-a)/s was in the three cases:
% 1/0.3 = 3.3333,  1/0.4 = 2.5,  1/0.7 = 1.4286
```

step increment

`x = a : s : b;`

the number of subintervals within [a,b] is obtained by rounding **`(b-a)/s`**, down to the nearest integer,

```
N = floor((b-a)/s);
```

`length(x)` is equal to **`N+1`**

```
x(n) = a + s*(n-1),
n = 1,2,...,N+1
```

Note: MATLAB array indices always start with 1
and may not be 0 or negative

exception:
logical indexing,
discussed later

```
>> x = [ 2,    5,   -6,   10,    3,    4 ];
      x(1), x(2), x(3), x(4), x(5), x(6)
```

Other languages, such as C/C++ and Fortran, allow indices to start at 0. For example, the same array would be declared/defined in C as follows:

```
double x[6] = { 2,    5,   -6,   10,    3,    4 };
              x[0], x[1], x[2], x[3], x[4], x[5]
```

accessing array entries:

```
>> x = [2, 5, -6, 10, 3, 4]
x =
     2     5    -6    10     3     4

>> length(x)     % length of x, see also size(x)
ans =
     6

>> x(1)          % first entry
ans =
     2

>> x(3)          % third entry
ans =
    -6

>> x(end)        % last entry - need not know length
ans =
     4
```

accessing array entries:

```
>> x(end-3:end)            % x = [2, 5, -6, 10, 3, 4]
ans =
     -6    10     3     4        % last four

>> x(3:5)                  % list third-to-fifth entries
ans =
    -6    10     3

>> x(1:3:end)              % every third entry
ans =
     2    10

>> x(1:2:end)              % every second entry
ans =
     2    -6     3
```

accessing array entries:

```
>> x = [2, 5, -6, 10, 3, 4];

>> x(end:-1:1)     % list backwards, same as fliplr(x)
ans =
     4    3    10    -6     5     2

>> x([3,1,5])        % list [x(3),x(1),x(5)]
ans =
     -6     2     3

>> x(end+3) = 8
x =
    2     5    -6    10     3     4     0     0     8
```

automatic memory re-allocation

automatic memory allocation and de-allocation:

```
>> clear x

>> x(3) = -6
x =
     0     0    -6

>> x(6) = 4
x =
     0     0    -6     0     0     4

>> x(end) = []                        % delete last entry
x =
     0     0    -6     0     0

>> x = [2, 5, -6, 10, 3, 4];
>> x(3)=[]                            % delete third entry
x =
     2     5     10     3     4
```

pre-allocation

```
>> clear x
>> x = zeros(1,6)          % 1x6 array of zeros
x =
     0     0     0     0     0     0

>> x = zeros(6,1)          % 6x1 array of zeros
x =
     0
     0
     0
     0
     0
     0
```

```
>> help zeros
>> help ones
```

illustrating dynamic allocation & pre-allocation

```
clear x;
for k=[3,7,10]
   x(k) = 3 + 0.1*k;
   disp(x);
end
```

```
% k runs successively through
% the values of [3,7,10]
% diplay current vector x
```

```
    0.0   0.0   3.3
    0.0   0.0   3.3   0.0   0.0   0.0   3.7
    0.0   0.0   3.3   0.0   0.0   0.0   3.7   0.0   0.0   4.0
```

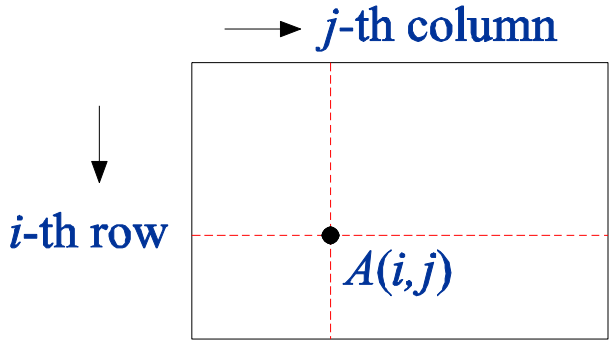
```
x = zeros(1,10);
for k=[3,7,10]
   x(k) = 3 + 0.1*k;
   disp(x);
end
```

```
% pre-allocate x to length 10
```

```
    0.0   0.0   3.3   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   3.3   0.0   0.0   0.0   3.7   0.0   0.0   0.0
    0.0   0.0   3.3   0.0   0.0   0.0   3.7   0.0   0.0   4.0
```

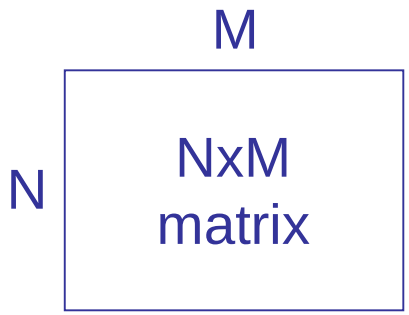
defining matrices

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$



matrix indexing convention

```
>> A = [1 2 3; 2 0 4; 0 8 5]
A =
     1     2     3
     2     0     4
     0     8     5
```




```
>> size(A)                % [N,M] = size(A), NxM matrix
ans =
     3     3
```

column dimension = no. rows

row dimension = no. columns

accessing matrix elements

```
>> A(1,1)        % 11 matrix element
ans =
     1
```

$$A = \begin{bmatrix} \textcircled{1} & 2 & 3 \\ 2 & 0 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

```
>> A(2,3)        % 23 matrix element
ans =
     4
```

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & \textcircled{4} \\ 0 & 8 & 5 \end{bmatrix}$$

```
>> A(:,2)        % second column
ans =
     2
     0
     8
```

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

```
>> A(3,:)         % third row
ans =
     0     8     5
```

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

concatenating columns

```
>> A = [1 2 3; 2 0 4; 0 8 5]
A =
     1     2     3
     2     0     4
     0     8     5
```

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

```
>> A(:)      % concatenate columns
ans =
     1
     2
     0
     2
     0
     8
     3
     4
     5
```

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

column-wise indexing

A(6) ← 8

A(9) ← 5

concatenating rows

```
B = A'; B(:)
```

see also the built-in functions:

sub2ind, ind2sub

building a matrix column-wise

```
>> A = zeros(3,3);

>> A(:) = [1 2 0  2 0 8  3 4 5]

A =
     1     2     3
     2     0     4
     0     8     5
```

define desired size

enter elements in a row (or column)

elements are re-arranged column-wise

sub-matrices

```
A = [ 2     4     1     3     5
      8     6     7     4     9
      3     2     5     2     1
      5     6     1     8     4 ];
```

```
A(3:4, 2:4)
ans =
     2     5     2
     6     1     8
```

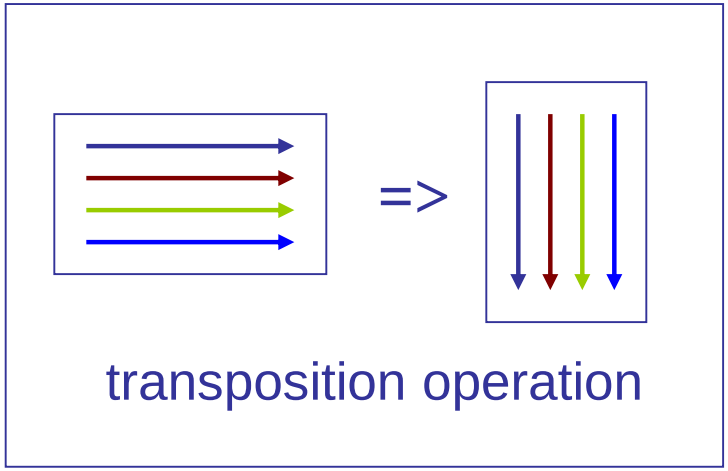
$$A = \begin{bmatrix} 2 & 4 & 1 & 3 & 5 \\ 8 & 6 & 7 & 4 & 9 \\ 3 & 2 & 5 & 2 & 1 \\ 5 & 6 & 1 & 8 & 4 \end{bmatrix}$$

```
A(1:3, [1,5])
ans =
     2     5
     8     9
     3     1
```

$$A = \begin{bmatrix} 2 & 4 & 1 & 3 & 5 \\ 8 & 6 & 7 & 4 & 9 \\ 3 & 2 & 5 & 2 & 1 \\ 5 & 6 & 1 & 8 & 4 \end{bmatrix}$$

transposing a matrix

```
>> A = [1 2 3 4; 2 0 5 6; 0 8 7 9]    % size 3x4
A =
     1     2     3     4
     2     0     5     6
     0     8     7     9

>> A'                                  % size 4x3
ans =
     1     2     0
     2     0     8
     3     5     7
     4     6     9
```



adding / deleting rows or columns

```
>> A = [1 2 3; 2 0 4; 0 8 5]

>> A(5,:) = [7 8 9]      % add a fifth row
A =
     1     2     3
     2     0     4
     0     8     5
     0     0     0
     7     8     9
```

4th row is automatically allocated

```
>> A(:,2) = []           % delete second column
A =
     1     3
     2     4
     0     5
     0     0
     7     9
```

[ ] denotes an empty 0x0 matrix

alternatively, redefine A by omitting its second column:

```
>> A = A(:,[1,3]);
```

replacing rows or columns

```
>> A = [1 2 3; 2 0 4; 0 8 5]
A =
     1     2     3
     2     0     4
     0     8     5

>> A(:,2) = [20 30 40]'    % replace second column
A =
     1    20     3
     2    30     4
     0    40     5

>> A(3,:) = [50 60 70]     % replace third row
A =
     1    20     3
     2    30     4
    50    60    70
```

inserting rows or columns

```
>> A = [1 2 3; 2 0 4; 0 8 5]
A =
     1     2     3
     2     0     4
     0     8     5

% insert new column between columns 2 & 3
A = [A(:,1:2), [10 20 30]', A(:,3)]
A =
     1     2    10     3
     2     0    20     4
     0     8    30     5

% insert new row between rows 1 & 2
A = [A(1,:); [60 70 80 90]; A(2:3,:)]
A =
     1     2    10     3
    60    70    80    90
     2     0    20     4
     0     8    30     5
```

concatenating matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

```
>> A = [1 2; 3 4];
>> B = [5 6; 7 8];

>> C = [A, B]
C =
     1     2     5     6
     3     4     7     8
```

A,B must have same number of rows

```
>> C = [A; B]
C =
     1     2
     3     4
     5     6
     7     8
```

A,B must have same number of columns

appending columns or rows

```
>> A = [1 2; 3 4; 5 6];
>> b = [7; 7; 7];
>> c = [8 8 8]';

>> B = [A,b,c]
B =
     1     2     7     8
     3     4     7     8
     5     6     7     8

>> C = [b,A,c]
C =
     7     1     2     8
     7     3     4     8
     7     5     6     8
```

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 8 \\ 8 \\ 8 \end{bmatrix}$$

```
>> D = [A; [7 7];]
>> E = [[8 8]; A]
D =
     1     2
     3     4
     5     6
     7     7
E =
     8     8
     1     2
     3     4
     5     6
```

special matrices

```
eye(3)    % 3x3 identity matrix
ans =
     1     0     0
     0     1     0
     0     0     1
```

```
>> help eye
>> help zeros
>> help ones
```

```
zeros(3)   % 3x3 matrix of zeros
ans =
     0     0     0
     0     0     0
     0     0     0
```

```
ones(3)    % 3x3 matrix of ones
ans =
     1     1     1
     1     1     1
     1     1     1
```

general usage:

eye(N,M)
zeros(N,M)
ones(N,M)

see also:

rand(N,M)
randn(N,M)
randi(I,N,M)

for more information on elementary matrices see:

```
>> help elmat

  Elementary matrices and matrix manipulation.

  Elementary matrices.
    zeros         - Zeros array.
    ones          - Ones array.
    eye           - Identity matrix.
    repmat        - Replicate and tile array.
    linspace      - Linearly spaced vector.
    logspace      - Logarithmically spaced vector.

    etc.
```

diagonals

```
>> A = [1 2 3; 4 5 6; 7 8 9]
A =
     1     2     3
     4     5     6
     7     8     9

>> d = diag(A)          % main diagonal
d =
     1
     5
     9

>> d = diag(A,-1)       % first sub-diagonal
d =
     4
     8
```

```
>> help diag
```

`diag(diag(A))` = what does it do?

how to make a diagonal matrix

```
>> d = [4 5 6];          % or, column d = [4 5 6]';

A = diag(d)              % d is main diagonal
A =
     4     0     0
     0     5     0
     0     0     6

>> d = [4 5];

A = diag(d,1)            % d is first upper-diagonal
A =
     0     4     0
     0     0     5
     0     0     0
     0     0     0
```

```
>> A = [1 2; 3 4]; B = [5 6; 7 8];
>> C = [9 8 7; 6 5 4; 3 2 1];

>> blkdiag(A,B)
ans =
     1     2     0     0
     3     4     0     0
     0     0     5     6
     0     0     7     8
```

how to make block-diagonal matrices

matrix dimensions expand as necessary

```
>> blkdiag(A,B,C)
ans =
     1     2     0     0     0     0     0
     3     4     0     0     0     0     0
     0     0     5     6     0     0     0
     0     0     7     8     0     0     0
     0     0     0     0     9     8     7
     0     0     0     0     6     5     4
     0     0     0     0     3     2     1
```

replicating matrices – using repmat

```
>> A=[1 2; 3 4]
A =
     1     2
     3     4
```

repmat works also with other data types, such as strings or cell arrays

```
>> repmat(A,3,4)
ans =
     1     2     1     2     1     2     1     2
     3     4     3     4     3     4     3     4
     1     2     1     2     1     2     1     2
     3     4     3     4     3     4     3     4
     1     2     1     2     1     2     1     2
     3     4     3     4     3     4     3     4
```

```
>> s = repmat('%7.4f   ', 1, 4)
s =
%7.4f   %7.4f   %7.4f   %7.4f
```

← replicated string

reshaping a matrix or a vector

B = reshape(A,P,Q)

reshapes an NxM matrix into a PxQ matrix (must have PxQ=NxM)

B is formed column-wise from the elements of A

```
>> a = [1 2 3 4 5 6];

>> reshape(a,2,3)
ans =
     1     3     5
     2     4     6

>> reshape(a,3,2)
ans =
     1     4
     2     5
     3     6
```

```
>> reshape(a,6,1)
ans =
     1
     2
     3
     4
     5
     6
```

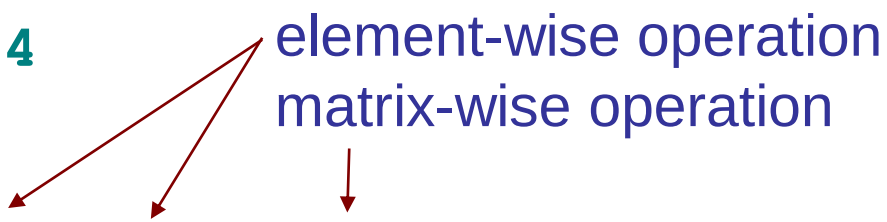
reshaping a matrix or a vector

```
A = [1    5    9
     2    6    5
     3    7    0
     4    8    4];

>> reshape(A,3,4)
ans =
     1    4    7    5
     2    5    8    0
     3    6    9    4


>> reshape(A,2,6)
ans =
     1    3    5    7    9    0
     2    4    6    8    5    4
```

```
>> reshape(A,6,2)
ans =
     1    7
     2    8
     3    9
     4    5
     5    0
     6    4
```

element-wise
matrix operations

```
>> A = [1 2; 3 4]
A =
     1     2
     3     4
```




```
>> [A, A.^2; 2.^A, A^2]          % form sub-blocks
ans =
     1     2     1     4
     3     4     9    16
     2     4     7    10          % note A.^2 ~= A^2
     8    16    15    22

>> B = 10.^A;
>> [B, log10(B)]
ans =
       10      100        1        2
     1000    10000        3        4
```

element-wise matrix operations

```
>> A=[1 4; 8 2], B=[1 2; 2 1]
A =
     1     4
     8     2
B =
     1     2
     2     1

>> A./B
ans =
     1     2
     4     2

>> A.\B
ans =
    1.0000    0.5000
    0.2500    0.5000
```

But note the matrix operations:

```
>> sym(A/B)   % A*inv(B)
ans =
   [  7/3, -2/3]
   [ -4/3, 14/3]

>> A\B        % inv(A)*B
ans =
    0.2   0.0
    0.2   0.5
```

element-wise matrix operations

```
>> A=[1 4; 8 2], B=[1 2; 2 1]
A =
     1     4
     8     2
B =
     1     2
     2     1

>> A.*B
ans =
     1     8
    16     2

>> A.^B
ans =
     1    16
    64     2

>> B.^A
ans =
     1    16
   256     1
```

functions of matrices

```
>> X = [pi/2, pi/3; pi/4, pi/8]
X =
    1.5708    1.0472
    0.7854    0.3927

>> sin(X)
ans =
    1.0000    0.8660
    0.7071    0.3827

>> sin(sym(X))
ans =
[          1,                3^(1/2)/2]
[ 2^(1/2)/2, (2 - 2^(1/2))^(1/2)/2]
```

many functions operate element-wise on matrices
e.g., trig, exp, log functions

others operate column-wise
e.g., min, max, sort, diff, mean, std, median, sum, cumsum, prod, cumprod

functions of matrices

functions that operate column-wise can also operate row-wise by using a second argument

```
A = [8      5      8
     9      1      3
     2      4      5
     6      2      2];

>> sum(A)
ans =
    25    12    18

>> cumsum(A)% comulative sum
ans =
     8     5     8
    17     6    11
    19    10    16
    25    12    18
```

```
>> sum(A,2)
ans =
    21
    13
    11
    10

>> cumsum(A,2)
ans =
     8    13    21
     9    10    13
     2     6    11
     6     8    10
```

functions of matrices

```
A = [8     5     8
     9     1     3
     2     4     5
     6     2     2];
```

```
>> mean(A), mean(A,2)
ans =
    6.25   3.00   4.50
ans =
    7.0000
    4.3333
    3.6667
    3.3333
```

means computed down each column

means computed across rows

```
>> [m,i]=min(A)
m =
     2     1     2
i =
     3     2     4
```

```
>> [m,i]=min(A,[],2);
>> [m,i]
ans =
     5     2
     1     2
     2     1
     2     2
```

min, max require a sightly different syntax for row-wise operation, similarly for **diff,std**

functions of matrices

```
A = [8     5     8
     9     1     3
     2     4     5
     6     2     2];
```

```
>> fliplr(A)
ans =
     8     5     8
     3     1     9
     5     4     2
     2     2     6
```

```
>> flipud(A)
ans =
     6     2     2
     2     4     5
     9     1     3
     8     5     8
```

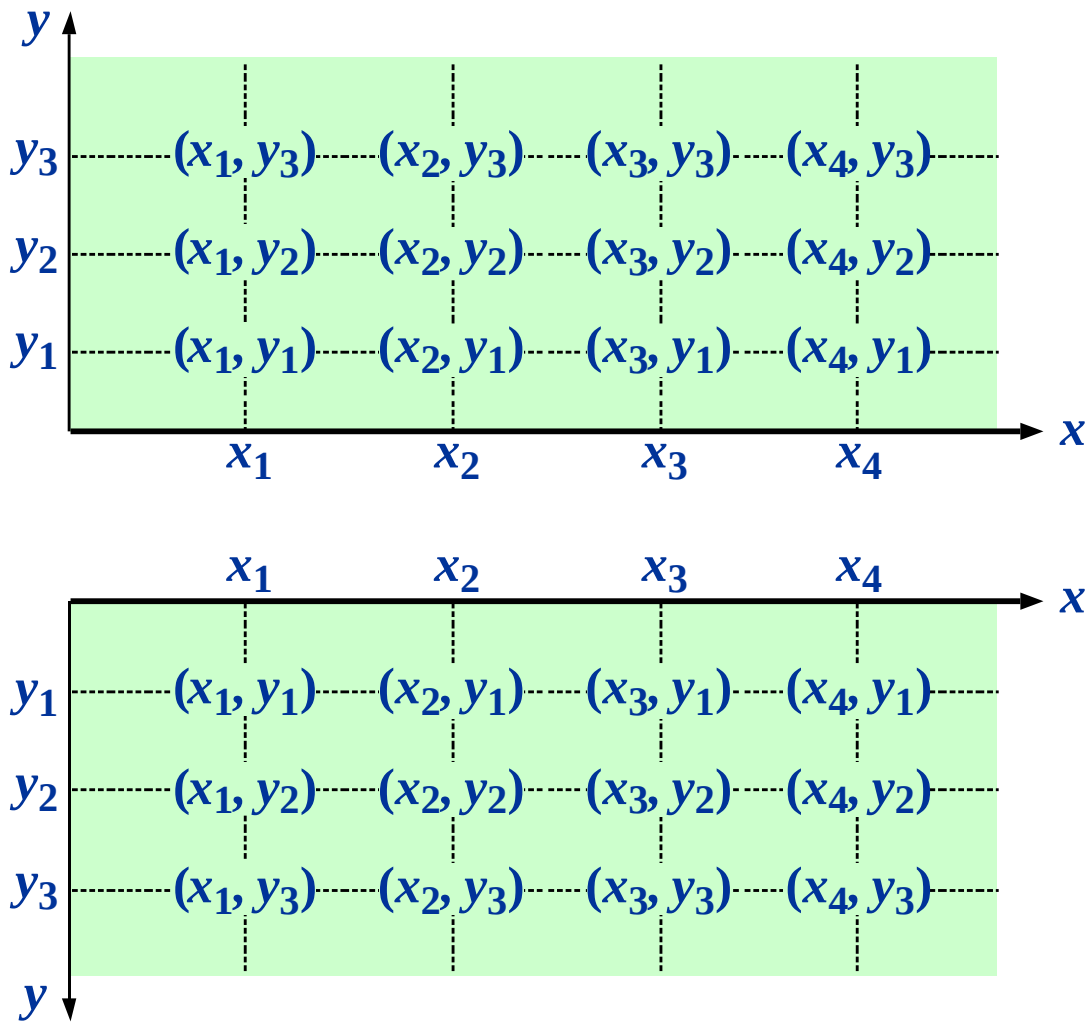
```
>> rot90(A)
ans =
     8     3     5     2
     5     1     4     2
     8     9     2     6
```

rotate by 90 degrees

reverse each row

reverse each column

`flipud(rot90(A))` and **`rot90(fliplr(A))`** are the same as **`A'`**



meshgrid function

a function of two variables,
$z = f(x,y)$
defines a surface

meshgrid

```
x = [x1,  x2,  x3,  x4]
y = [y1,  y2,  y3]

[X,Y] = meshgrid(x,y)
```

$$X = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_1 & x_2 & x_3 & x_4 \\ x_1 & x_2 & x_3 & x_4 \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 & y_1 & y_1 & y_1 \\ y_2 & y_2 & y_2 & y_2 \\ y_3 & y_3 & y_3 & y_3 \end{bmatrix}$$

meshgrid
ndgrid

```
>> x = [1 2 3 4];   % N-dim
>> y = [5 6 7];     % M-dim

>> [X,Y] = meshgrid(x,y)
X =
   1   2   3   4
   1   2   3   4
   1   2   3   4
Y =
   5   5   5   5
   6   6   6   6
   7   7   7   7
```

x is replicated row-wise

y is replicated column-wise

X,Y have size MxN

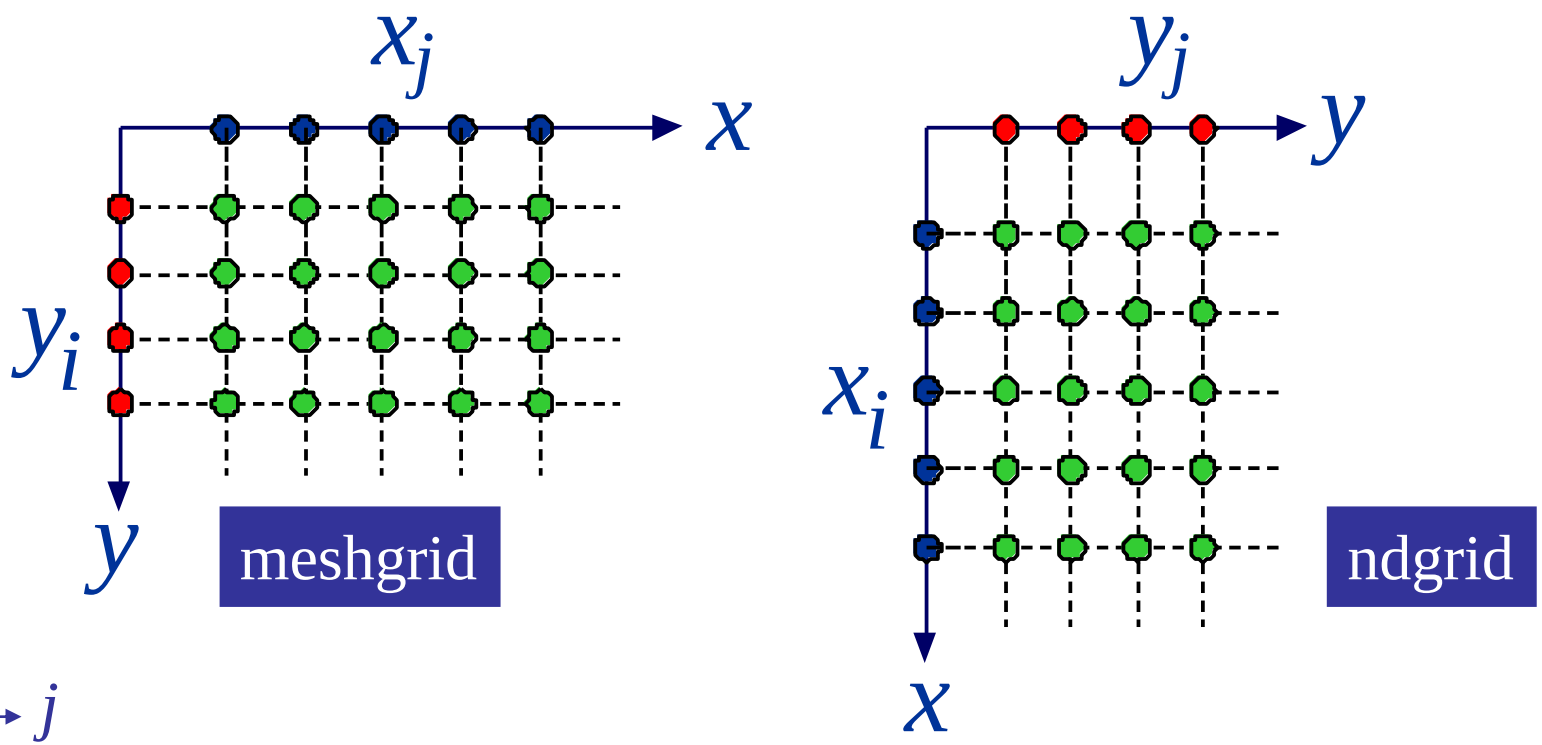
```
[Y,X] = meshgrid(y,x)
```

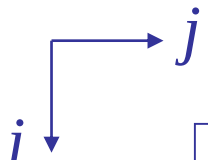
equivalent

```
>> [X,Y] = ndgrid(x,y)
X =
    1   1   1
    2   2   2
    3   3   3
    4   4   4
Y =
    5   6   7
    5   6   7
    5   6   7
    5   6   7
```

x is replicated column-wise, and **y**, row-wise

X,Y have size NxM





[X,Y] = meshgrid(x,y)

x = rows
y = columns

[X,Y] = ndgrid(x,y)

x = columns
y = rows

equivalent

[Y,X] = meshgrid(y,x)

[Y,X] = ndgrid(y,x)