Introduction to Applied Scientific Computing using MATLAB

Mohsen Jenadeleh

In this lecture, slides from MIT, Rutgers and Waterloo University are used to form the lecture slides

Topics

Relational and logical operators
Precedence rules
Logical indexing

`find` function

Program flow control

`if` – statements
`switch` – statements

Examples:
piece-wise functions, unit-step function, indicator functions, sinc function

Choose Symbolic or Numeric Arithmetic

sin(π)	Symbolic	Variable Precision	Double Precision
	a = sym(pi) sin(a)	b = vpa(pi) %vpa(pi,d) sin(b)	pi sin(pi)
	a = pi ans = 0	b = 3.14159265358979323846264338327 95 ans = - 3.21010830131003960695471458835 68e-40	ans = 3.1416 ans = 1.2246e-16

Round-Off Errors	No, finds exact results	Yes, magnitude depends on precision used (32 defult)	Yes, has 16 digits of precision
Speed	slowest	Faster, depends on precision used	faster
Memory Usage	Greatest	Adjustable, depends on precision used	Least

Relational and Logical Operators

Relational and logical functions

```
find, logical, true, false, any, all

ischar, isequal, isfinite, isinf, isinteger
islogical, isnan, isreal
```

```
>> doc is*           % list of all 'is' functions
>> help logical      % convert to logical
>> help true         % logical 1
>> help false        % logical 0
>> help relop        % relational operators (&,|,…)
>> help ops          % same as help /
>> help find         % indices of non-zero elements
```

```
>> help precedence %Operator Precedence in MATLAB.
```

Relational Operators

`>> help relop`

`==`	equal
`~=`	not equal
`<`	less than
`>`	greater than
`<=`	less than or equal
`>=`	greater than or equal

Logical Operators

`&`	logical AND,	e.g., A&B, A,B=expressions
`|`	logical OR,	e.g., A|B
`~`	logical NOT,	e.g., ~A
`&&`	logical AND for scalars w/ short-circuiting	
`||`	logical OR for scalars w/ short-circuiting	
`xor`	exclusive OR,	e.g., xor(A,B)
`any`	true if any elements are non-zero	
`all`	true if all elements are non-zero	

Operator Precedence in MATLAB (from highest to lowest):

1. transpose (.'), power (.^), conjugate transpose ('), matrix power (^)
2. unary plus (+), unary minus (-), logical negation (~)
3. multiplication (.*), right division (./), left division (.\), matrix multiplication (*), matrix right division (/), matrix left division (\)
4. addition (+), subtraction (-)
5. colon operator (:)
6. less than (<), less than or equal to (<=), greater than (>), greater than or equal to (>=), equal to (==), not equal to (~=)
7. element-wise logical AND (&)
8. element-wise logical OR (|)
9. short-circuit logical AND (&&)
10. short-circuit logical OR (||)

```
>> help precedence
```

```
>> a = [1, 0, 2, -3, 7];
>> b = [3, 4, 2, -1, 7];

>> a == b
ans =
     0     0     1     0     1

>> k = a == b       % clearer notation, k = (a==b)
ans =
     0     0     1     0     1

>> class(k)
ans =
     logical

>> a(k)        <-- logical indexing
ans =
     2     7
```

```
>> a = [1, 0, 2, -3, 7];
>> b = [3, 4, 2, -1, 7];

>> a == b
ans =
     0     0     1     0     1

>> k = a == b      % clearer notation, k = (a==b)
ans =
     0     0     1     0     1

>> i = find(a==b)
i =
     3     5

>> a(i)
ans =
     2     7
```

using find

regular indexing

a(a==b), a(find(a==b))

```
>> a = [1, 0, 2, -3, 7];
>> b = [3, 4, 2, -1, 7];

>> a == b
ans =
     0     0     1     0     1
>> a ~= b
ans =
     1     1     0     1     0

>> i = find(a~=b)
i =
     1     2     4

>> a(i),b(i)
ans =
     1     0    -3
ans =
     3     4    -1
```

```
>> a = [1, 0, 2, -3, 7];

>> ~a
ans =
     0     1     0     0     0

>> a==0
ans =
     0     1     0     0     0

>> i = find(~a)
i =
     2
```

finds the zero entries of **a**

```
>> a = [1, 0, 2, -3, 7];

>> a~=0
ans =
     1     0     1     1     1
>> ~~a
ans =
     1     0     1     1     1

>> logical(a)
ans =
     1     0     1     1     1

>> i = find(a)
i =
     1     3     4     5

>> a(find(a))
ans =
     1     2    -3     7
```

finds the non-zero entries of **a**

```
>> a = [1, 0, 2, -3, 7];
>> b = [3, 4, 2, -1, 7];
```

case 1: both **a,b** are vectors

```
>> a<b, a>=b
```

a,b are compared element-wise

```
ans =
     1     1     0     1     0
ans =
     0     0     1     0     1

>> i = find(a<b)
i =
     1     2     4

>> a(a<b), a(find(a<b))
ans =
     1     0    -3
ans =
     1     0    -3
```

```
>> a = [1, 0, 2, -3, 7];
>> b = 1;
```

case 2: **a,b** are vector,scalar

```
>> a>=b
```

compare each element of **a** to the scalar **b**

```
ans =
      1      0      1      0      1

>> i = find(a>=b)
i =
      1      3      5

>> a(a>=b), a(find(a>=b)), a(a<b)
ans =
      1      2      7
ans =
      1      2      7
ans =
      0     -3
```

```
>> a = [1, 0, 2, -3, 7];
>> b = [3, 4, 2, -1, 7];

>> a>=1
ans =
     1     0     1     0     1

>> b<=2
ans =
     0     0     1     1     0

>> a>=1 & b<=2          % logical AND
ans =
     0     0     1     0     0

>> a>=1 | b<=2          % logical OR
ans =
     1     0     1     1     1
```

logical operations

```
>> a = [1, 3, 4, -3, 7];
>> k = (a>=2), i = find(a>=2)
k =
     0     1     1     0     1
i =
     2     3     5

>> a(i), a(k)
ans =
     3     4     7
ans =
     3     4     7

>> n = [0 1 1 0 1]
>> a(n)
??? Subscript indices must either be real
positive integers or logicals.

% but note, a(logical(n)) works
```

logical indexing

class(k) is logical

logical indexing

a(a>=2)

class(n) is double, but n==k is true

```
>> A = [3 4 nan; -5 inf 2]
A =
     3     4   NaN
    -5   Inf     2

>> k = isfinite(A)
k =
     1     1     0
     1     0     1

>> A(k)        % listed column-wise
ans =
     3
    -5
     4
     2

>> A(~k)=0   % set non-finite
A =          % entries to zero
     3     4     0
    -5     0     2
```

more on
logical indexing

```
>> find(k)
ans =
     1
     2
     3
     6
```

```
>> [i,j] = find(k)
[i,j] =
     1   1
     2   1
     1   2
     2   3
```

```
>> A = [3 4 0; -5 5 2]
A =
     3     4     0
    -5     5     2

>> A>2
ans =
     1     1     0
     0     1     0

>> k = find(A>2)
k =
     1
     3
     4
```

```
>> [i,j] = find(A>2);
[i,j] =
         1   1
         1   2
         2   2

>> A(find(A>2))
ans =
      3
      4
      5
```

```
>> K = ['1'    '2'    '3'
        '4'    '5'    '6'
        '7'    '8'    '9'
        '*'    '0'    '#'];
```

find can also be applied to a matrix of characters, e.g., the keypad matrix from week-3

```
>> K=='8'
ans =
     0     0     0
     0     0     0
     0     1     0
     0     0     0
```

compares every element of K with '8'

find the location of the correct element of K

```
>> [i,j] = find(K=='8')
i =
     3
j =
     2
```

i,j matrix indices of the location of '8'

```
>> q = find(K=='8')
q =
     7
```

q is the column-wise index of '8' in K

```
A = [9   9   2       B = [7   1   7
     2   5   4            3   4   8
     9   8   9];          9   4   2];
```

```
>> A<B
ans =
     0   0   1
     1   0   1
     0   0   0
```

```
>> find(A<B)
ans =
     2
     7
     8
```

```
[i,j]=find(A<B)
i =   j =
   2     1
   1     3
   2     3
```

```
>> A==9
ans =
     1   1   0
     0   0   0
     1   0   1
```

```
>> find(A==9)
ans =
     1
     3
     4
     9
```

```
>> A(A==9)=-9
A =
    -9   -9    2
     2    5    4
    -9    8   -9
```

any, all

```
A = [9   9   2        B = [7   1   7
     2   5   4             3   4   8
     9   8   9];           9   4   2];
```

```
any(A==2)
ans =
     1   0   1

any(A==2,2)
ans =
     1
     1
     0
```

```
all(A>B)
ans =
     0   1   0

all(A>B,2)
ans =
     0
     0
     0
```

```
A==B
ans =
     0   0   0
     0   0   0
     1   0   0

any(A==B)
ans =
     1   0   0

any(any(A==B))
ans =
     1
```

```
all(all(A==B));
```

any,all operate column-wise, or, row-wise with extra argument

```
>> A = [36 -4 9; 16 9 -25], B = A;

A =
    36    -4     9
    16     9   -25


>> k = (B>=0)

k =
     1     0     1
     1     1     0


>> B(k) = sqrt(B(k));
>> B(~k) = -sqrt(-B(~k))

B =
     6    -2     3
     4     3    -5
```

Example:
take square-roots of the absolute values, but preserve the signs

Comparing Strings

Strings are arrays of characters, so the condition **s1==s2** requires both **s1** and **s2** to have the same length

```
>> s1 = 'short'; s2 = 'shore';
>> s1==s1
ans =
     1     1     1     1     1

>> s1==s2
ans =
     1     1     1     1     0

>> s1 = 'short'; s2 = 'long';
>> s1==s2
??? Error using ==> eq
Matrix dimensions must agree.
```

Comparing Strings

Use **strcmp** to compare strings of unequal length, and get a binary decision

```
>> s1 = 'short'; s2 = 'shore';
>> strcmp(s1,s1)
ans =
     1

>> strcmp(s1,s2)
ans =
     0

>> s1 = 'short'; s2 = 'long';
>> strcmp(s1,s2)
ans =
     0
```

```
>> doc strcmp
>> doc strcmpi
```

case-insensitive

Use **isequal** to compare the contents of matrices or arrays and get a binary decision

Program Flow Control

Program flow is controlled by the following control structures:

1. for . . . end `% loops`
2. while . . . end
3. break, continue
4. if . . . end `% conditionals`
5. if . . . else . . . end
6. if . . . elseif . . . else . . . end
7. switch . . . case . . . otherwise . . . end
8. return

for-loops and conditional ifs are by far the most commonly used control stuctures

if - statements

three forms of **if** statements

```
if condition
   statements ...
end
```

```
if condition
   statements ...
else
   statements ...
end
```

```
if condition1
   statements ...
elseif condition2
   statements ...
elseif condition3
   statements ...
else
   statements ...
end
```

several **`elseif`** statements may be present,

`elseif` does not need a matching **`end`**

Example

```
>> x = 1;
>> % x = 0/0
>> % x = 1/0

if isinf(x),
   disp('x is infinite');
elseif isnan(x),
   disp('x is not-a-number');
else
   disp('x is finite number');
end

x is finite number
% x is not-a-number
% x is infinite
```

switch - statements

```
switch expression0

  case expression1
      statements ...

  case expression2
      statements ...

  otherwise
      statements ...

end
```

expression0 is evaluated first, and if its value matches any of the cases expression1, expression2, … , then the corresponding case statements are executed

several case statements may be present

expression comparison rules:

numbers: `isequal(expression0, expression1)`

strings: `strcmp(expression0, expression1)`

```
x = [1, 4, -5, 3];

p = inf;
% p = 1;
% p = 2;

switch p
    case 1
        N = sum(abs(x));               % N = norm(x,1);
    case 2
        N = sqrt(sum(abs(x).^2));      % N = norm(x,2);
    case inf
        N = max(abs(x));               % N = norm(x,inf);
    otherwise
        N = sqrt(sum(abs(x).^2));      % N = norm(x,2);
end

>> N
N =
     5
```

equivalent calculation using the built-in function **norm**

Example: L_1, L_2, and L_∞ norms of a vector

$$\mathbf{x} = [x_1, x_2, \ldots, x_N]$$

$$\|\mathbf{x}\|_1 = \sum_{n=1}^{N} |x_n|$$

$$\|\mathbf{x}\|_2 = \sqrt{\sum_{n=1}^{N} |x_n|^2}$$

$$\|\mathbf{x}\|_\infty = \max(|x_1|, |x_2|, \ldots, |x_N|)$$

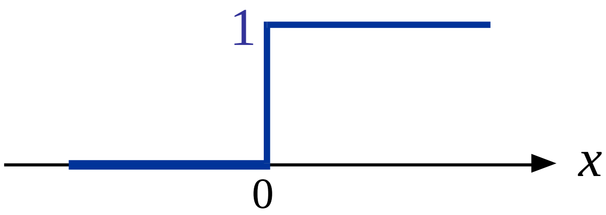
$$d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|$$

used as distance measure between two vectors or matrices

```
>> help norm    % vector and matrix norms
```

Example: unit-step function

$$u(x) = \begin{cases} 1, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

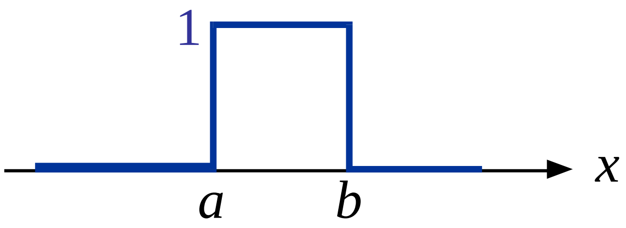



```
u = @(x) (x>=0);        % unit-step function
```

```
e.g., x =-3,-2,-1, 0, 1, 2, 3
    u(x)= 0, 0, 0, 1, 1, 1, 1
```

Example: indicator function

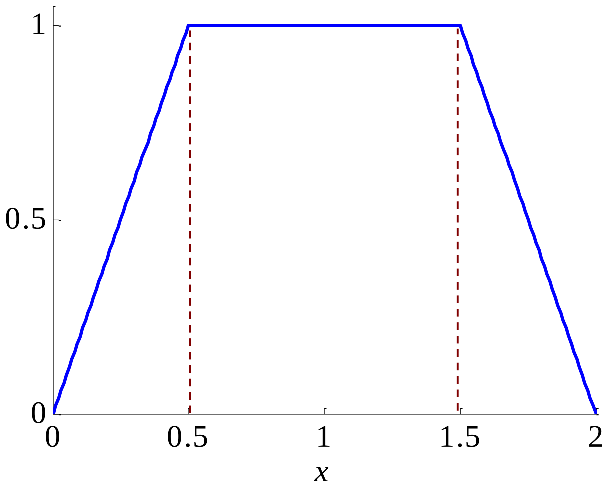
$$v(x,a,b) = u(x-a) - u(x-b)$$




```
v = @(x,a,b) u(x-a)-u(x-b);    % indicator
% v = @(x,a,b) (x>=a & x<b);   % alternative
```

Example: Defining piece-wise functions (method 1)

$$f(x) = \begin{cases} 2x, & 0 \le x \le 0.5 \\ 1, & 0.5 \le x \le 1.5 \\ 4 - 2x, & 1.5 \le x \le 2 \end{cases}$$



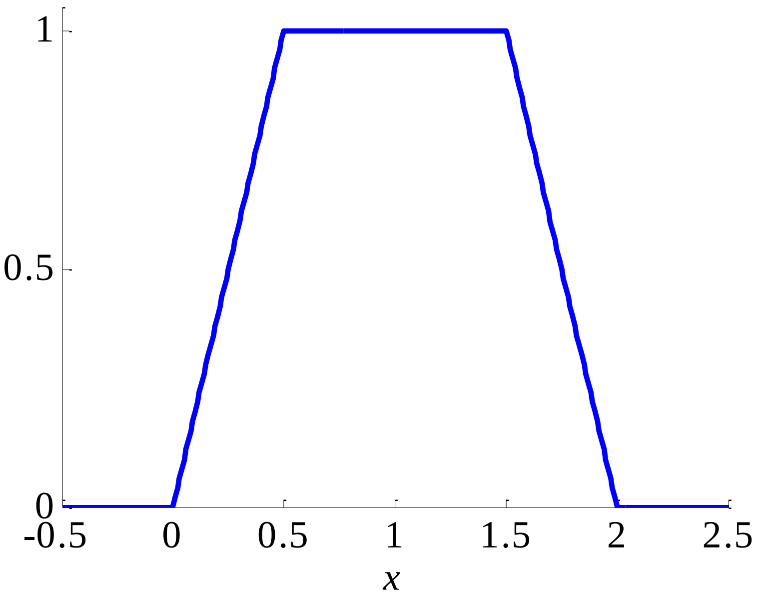
$$v(x, a, b) = \begin{cases} 1, & a \le x < b \\ 0, & \text{otherwise} \end{cases} = \text{(indicator function)}$$

$$f(x) = 2x\, v(x,\ 0,\ 0.5) + v(x,\ 0.5,\ 1.5) + (4 - 2x)\, v(x,\ 1.5,\ 2)$$

```
f = @(x) 2*x .* (x>=0 & x<0.5) + ...
                (x>=0.5 & x<1.5) + ...
      (4-2*x).* (x>=1.5 & x<2);

x = linspace(-0.5,2.5,301);

figure; plot(x,f(x), 'b-');
```

Anonymous Function

- is a function that is *not* stored in a program file
- can accept inputs and return outputs
- they can contain only a single executable statement.



Understanding the conditions `(x>=0 & x<0.5)`, etc.

```
x = [-0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2 ...
      0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0 ...
      1.1  1.2  1.3  1.4  1.5  1.6  1.7  1.8 ...
      1.9  2.0  2.1  2.2  2.3  2.4  2.5];

(x>=0 & x<0.5)
ans =
  0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

(x>=0.5 & x<1.5)
ans =
  0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0

(x>=1.5 & x<2)
ans =
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0
```

$$g(c) = \int_0^1 (x^2 + cx + 1)dx$$

```
g = @(c) (integral(@(x) (x.^2 + c*x + 1),0,1));
```

Write the integrand as an anonymous function,

```
@(x) (x.^2 + c*x + 1)
```

Evaluate the function from zero to one by passing the function handle to integral,

```
integral(@(x) (x.^2 + c*x + 1),0,1)
```

Supply the value for **c** by constructing an anonymous function for the entire equation

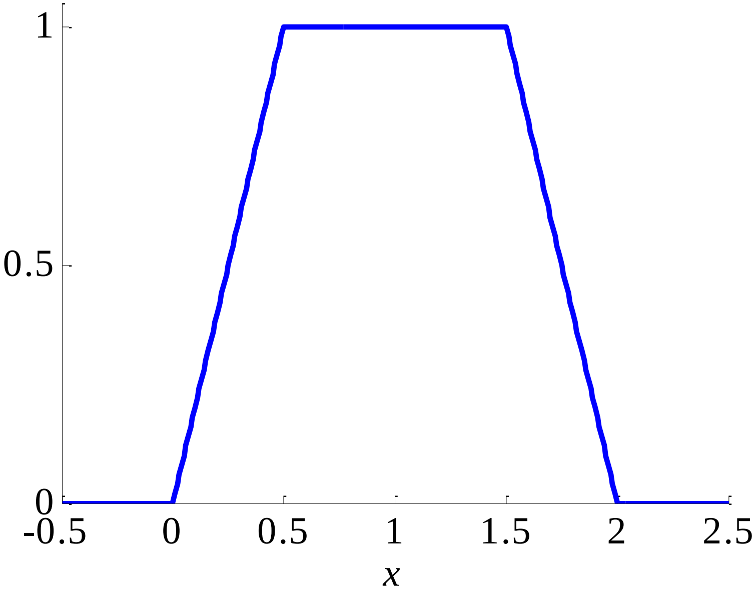

```
g = @(c) (integral(@(x) (x.^2 + c*x + 1),0,1));
```


The final function allows you to solve the equation for any value of c.

```
g(2)
ans = 2.3333
```

Using the indicator function

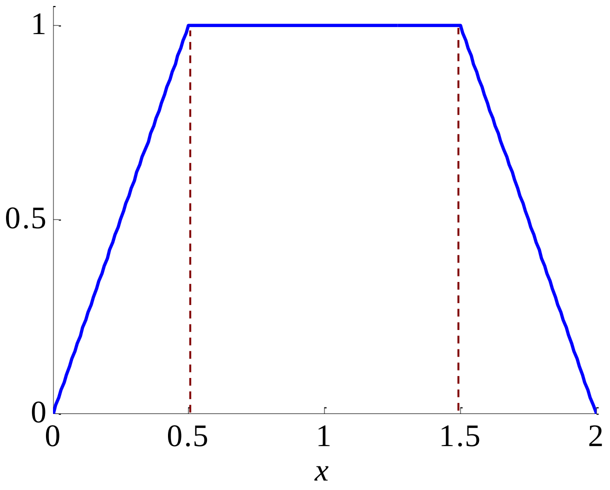
```
v = @(x,a,b) ((x>=a) & (x<b));

f = @(x) 2*x .* v(x, 0, 0.5) + ...
                v(x, 0.5, 1.5) + ...
      (4-2*x).* v(x, 1.5, 2);
```



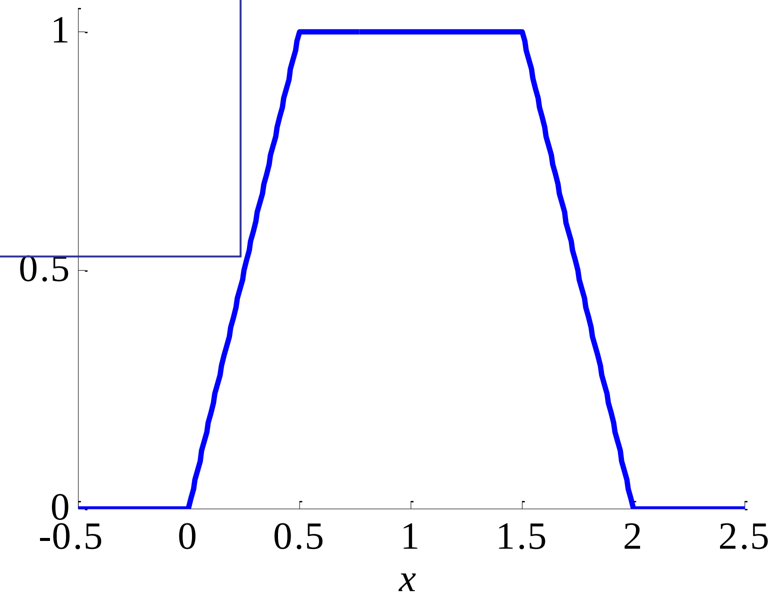
Example: Defining piece-wise functions (method 2)

x is a vector

```
function y = f(x)

y = zeros(size(x));

i1 = find(x>=0 & x<0.5);
y(i1) = 2*x(i1);

i2 = find(x>=0.5 & x<1.5);
y(i2) = 1;

i3 = find(x>=1.5 & x<2);
y(i3) = 4-2*x(i3);
```



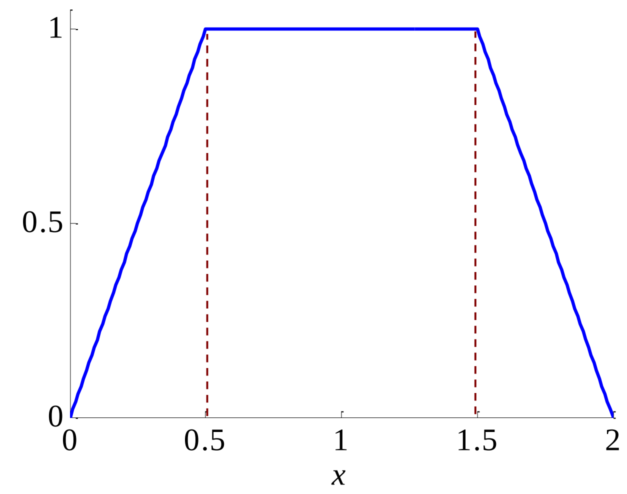
```
x = linspace(-0.5,2.5,301);
y = f(x);

figure; plot(x,y, 'b-');

axis([-0.5 2.5 0 1.2]);
xlabel('x-axis')
ylabel('y-axis')
xlim([-0.5 1])
ylim([0 2])
```



Example: Defining piece-wise functions (method 3)

x must be a scalar

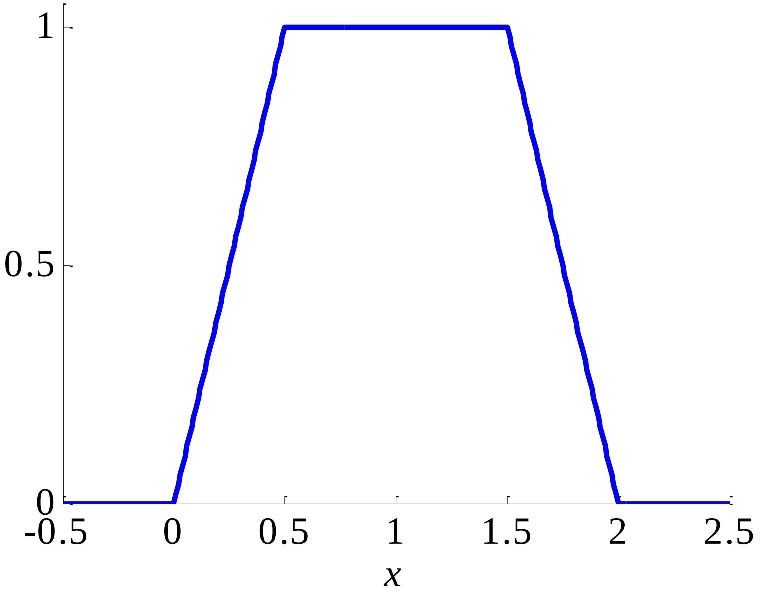
```
function y = f(x)

if x>=0 & x<0.5
    y = 2*x;
elseif x>=0.5 & x<1.5
    y = 1;
elseif x>=1.5 & x<2
    y = 4-2*x;
else
    y = 0;
end
```



pitfall: function produces wrong results if applied to a vector **x**, why?

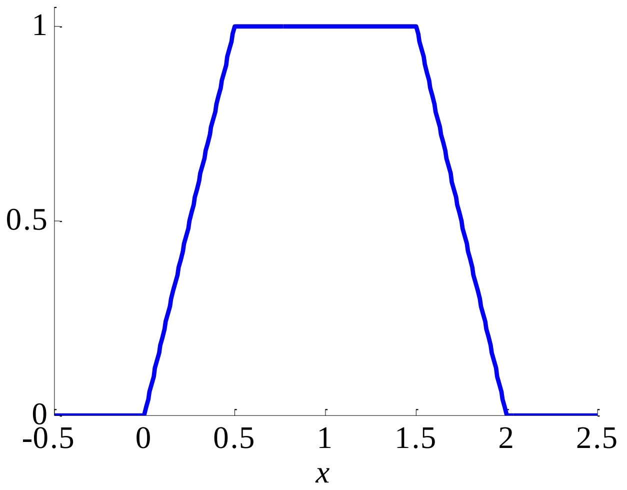
```
x = linspace(-0.5,2.5,301);

for n=1:length(x)
   y(n) = f(x(n));
end

figure; plot(x,y, 'b-');
yaxis(0,1.2, 0:0.5:1)
xaxis(-0.5,2.5, -0.5:0.5:2.5);
xlabel('\itx');
```

apply function separately to each element of **x**, instead of the whole **x**



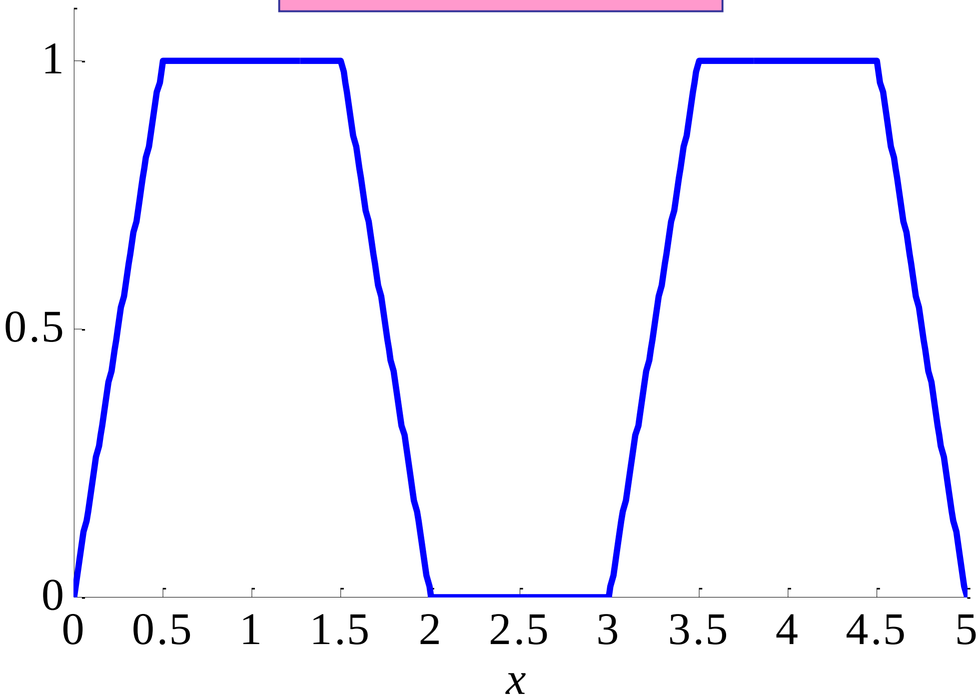
```
x = linspace(-0.5,2.5,301);

for n=1:length(x)
    if x(n)>=0 & x(n)<0.5
        y(n) = 2*x(n);
    elseif x(n)>=0.5 & x(n)<1.5
        y(n) = 1;
    elseif x(n)>=1.5 & x(n)<2
        y(n) = 4-2*x(n);
    else
        y(n) = 0;
    end
end

figure; plot(x,y, 'b-');
yaxis(0,1.2, 0:0.5:1)
xaxis(-0.5,2.5, -0.5:0.5:2.5);
xlabel('\itx');
```

direct implementation using if-elseif statements within a for-loop



```
f = @(x) 2*x .* (x>=0 & x<0.5) + ...
                (x>=0.5 & x<1.5) + ...
      (4-2*x).* (x>=1.5 & x<2);

x = linspace(0,10,501);

figure; plot(x,f(x)+f(x-3)+f(x-5), 'b-');
```

replicating f(x)



Example: Evaluating the sinc function

```
function y = my_sinc(x)

y = sin(pi*x)./(pi*x);

y(isinf(x)) = 0;

y(x==0) = 1;
```

generates **NaN**s for **x=inf** and **x=0**

fix **NaN** when **x=inf**

fix **NaN** when **x=0**

Note: built-in **sinc** function returns **NaN** when **x=inf**

```
x = [0, 0, inf, 0, nan];
y = sin(pi*x)./(pi*x)
y =
   NaN   NaN   NaN   NaN   NaN

isinf(x)
ans =
     0     0     1     0     0
y(isinf(x)) = 0
y =
   NaN   NaN     0   NaN   NaN

x==0
ans =
     1     1     0     1     0
y(x==0) = 1
y =
     1     1     0     1   NaN
```