Digital Signal Processing

WS 2017/18 Lab Sheet 12

Due date: Saturday, 03.02.2018

Exercise 1: Quantization — 7 Points

Estimate the quantization error of the signal $x[n] = A\cos\left(\frac{n}{10}\right)$ for different values of B with $X_m = 1$ fixed and A varying as follows:

a. Derive a formula for the $\mathrm{SN}R_Q$, defined as the average power over many samples of the signal divided by the estimate of the average power of the noise. (2)

b. Generate many samples of the signal using Matlab and compute and plot the $\mathrm{SNR_Q}$ as a function of $\frac{X_m}{\sigma_x}$ in the range $\left[\frac{1}{10}; 100\right]$ on a logarithmic scale for $B = 5, 7, \ldots, 15$. Explain the result. (5)

Exercise 2: DFT — 19 Points

a. Write the following two Matlab functions:

   i) `function [Xk] = dft(xn)` that takes a finite-duration sequence $x[n]$ and returns a complex DFT coefficient array $X[k]$ with $0 \leq n, k \leq N-1$. (2)

   ii) `function [xn] = idft(Xk)` that takes an array of complex DFT coefficients $X[k]$ and returns a finite duration sequence $x[n]$. (2)

b. Consider the periodic sequences

   i) $x[n] = \left(\frac{1}{2}\right)^n$ for $n = -2, \ldots, 3$ and $x[n]$ has the period 6, (3)

   ii) $x[n] = \sin(2\pi n/3)\cos(\pi n/2)$. (3)

   For both sequences, compute the complex DFT coefficients by hand and verify them using your Matlab function `dft` and the Matlab function `fft`. Compute an inverse Fourier transform on the complex DFT coefficients by using your Matlab function `idft` and the Matlab function `ifft`.

c. Modify your `dft` function, so that you can pass an additional parameter `k = k_min:k_max` to the function `[Xk] = dft(xn,k)`. The additional parameter `k` specifies the range of indices for which the output sequence of complex DFT coefficients is computed. (3)

d. Use your function and plot the real part, imaginary part, magnitude, and phase of the DFT coefficients of the following sequences for `k = -10:10`. Check for the symmetry properties of the DFT coefficients. (6)

   i) $x[n] = 3^{((n))_4}$,

   ii) $x[n] = \left(\frac{1}{2}\right)^{((n+2))_6 - 2}$,

   iii) $x[n] = \sin(2\pi n/3)\cos(\pi n/2)$.

Exercise 3: FT, DTFT, and DFT — 14 Points

a. The impulse response of a continuous LTI-system is

$$h_c(t) = \begin{cases} 1 & 0 \leq t < 4 \\ 0 & \text{else} \end{cases}.$$

   Compute its continuous Fourier transform $H_c(\Omega) = \int\limits_{t=-\infty}^{\infty} h_c(t) e^{-i\Omega t}\, \mathrm{d}t$ by hand. Plot the signal and its spectrum $|H_c(\Omega)|$ for $\Omega \in [0, 10]$ into two subfigures. (3)

b. A discrete version of this LTI-system has the impulse response (3)

$$h[n] = \begin{cases} 1 & 0 \leq n < 4 \\ 0 & \text{else} \end{cases}.$$

   Compute the DTFT $H(\omega)$ by hand and plot $h[n]$ and $|H(\omega)|$ into the subfigures.

c. A spectrum analyzer computes a 4-point DFT starting at $h[0]$. Compute its output $H_{\text{spec}}[k]$ by hand and using `dft`. Determine the impulse response $h_{\text{spec}}[n]$ of a system whose frequency response is the same as the output of the analyzer. Plot $h_{\text{spec}}[n]$ and $|H_{\text{spec}}[k]|$ into your subfigures. Take care of proper scaling of the discrete frequencies. (Use `stem` for discrete sequences!) (4)

d. Now, the spectrum analyzer is set to compute 8-point and 16-point DFTs. Does this help to approximate $H_c(\Omega)$? Use `dft` to compute the output and plot the results into your subfigures again. (4)

Maximal score: 40 Points