Digital Signal Processing

WS 2017 Lab Sheet 7

Due date: 16.12.2017

Exercise 1: Convolution and Parseval's Theorem 10 Points

Let $x[n]$ and $y[n]$ denote complex sequences and $X(e^{jw})$ and $Y(e^{jw})$ their respective Fourier transforms.

a. By using the convolution theorem and appropriate properties, determine, in terms of $x[n]$ and $y[n]$, the sequence, whose Fourier transform is $X(e^{j\omega})\,Y^*(e^{j\omega})$. (2)

b. Using the result in part a, show, that (4)

$$\sum_{n=-\infty}^{\infty} x[n]\,y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) Y^*\left(e^{j\omega}\right) \mathrm{d}\omega\,.$$

c. Using this equation, determine the numerical value of the sum (4)

$$\sum_{n=-\infty}^{\infty} \frac{\sin\left(\pi\frac{n}{4}\right)}{2\pi n}\frac{\sin\left(\pi\frac{n}{6}\right)}{5\pi n}$$

Exercise 2: DTFT and Convolution 8 Points

Using DTFT find the response of the system with impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$ to the input $x[n] = \left(\frac{1}{3}\right)^n u[n]$.

Exercise 3: Fourier transform 9 Points

Let $x[n]$ and $X(e^{jw})$ represent a sequence and its Fourier transform, respectively. Determine, in terms of $X(e^{j\omega})$, the transforms of $y_s[n]$, $y_d[n]$, and $y_e[n]$ as defined below. In each case, sketch the corresponding output Fourier transform $Y_s(e^{j\omega})$, $Y_d(e^{j\omega})$, and $Y_e(e^{j\omega})$, respectively for

$$X\left(e^{j\omega}\right) = 1 - \frac{|\omega|}{\pi},\ |\omega| \leq \pi\,.$$

a. Sampler:

$$y_s\left[n\right]=\begin{cases}x\left[n\right], & n\,\text{even}\\ 0, & n\,\text{odd}\end{cases}$$

(3)

b. Compressor:

$$y_d\left[n\right]=x\left[2n\right]$$

(3)

c. Expander:

$$y_e\left[n\right]=\begin{cases}x\left[\frac{n}{2}\right], & n\,\text{even}\\ 0, & n\,\text{odd}\end{cases}$$

(3)

Maximal score: 27 Points