Digital Signal Processing

WS 2017 Lab Sheet 1

Due date: 05.11.2017

Exercise 1: 10 Points

Simplify the following complex terms and give the result in both cartesian and polar form.

a. $z = \frac{2}{(1-\mathrm{j})(1+j)}$ (1)

b. $z = (1-j)^{43}$ (2)

c. $\frac{z-1}{z+1}$, $z \in \mathbb{C} \setminus \{-1\}$. (3)

d. $z = \frac{2-\mathrm{j}3}{5+j12}$ (2)

e. $z = 2\mathrm{e}^{-32\pi j/3}$ (2)

Exercise 2: 8 Points

a. Find an identity for $\sin(3\Phi)$ using $n = 3$ in De Moivre's formula. Write your identity in a way that involves only $\sin(\Phi)$ and $\sin^3(\Phi)$ if possible. (3)

b. Show the same as in a, but for $\cos(3\Phi)$ and use double angle formulas (2Φ) instead of De Moivre's formula. (3)

c. Show, that $\cos(\phi) = \frac{1}{2}\left(e^{j\phi} + e^{-\mathrm{j}\phi}\right)$ Find a similar expression for $\sin(\phi)$. (2)

Exercise 3: 8 Points

Solve the following equations for $z \in \mathbb{C}$.

a. $z^2 + 2z + 2 = 0$ (1)

b. $z^2 + 2jz = 1$ (1)

c. $z^3 = -8$ (2)

d. $z^3 = 8j$ (2)

e. $z^n = 1 - \mathrm{j}$, $n \in \mathbb{N}$ (2)

Maximal score: 26 Points

Matlab Introduction

Practise 1: 0 Points

Find a short expression, which creates the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 9 & 7 & 5 & 3 & 1 & -1 \\ 4 & 8 & 16 & 32 & 64 & 128 \end{pmatrix}$$

Hint: :-operator

Practise 2: 0 Points

Find a short expression, which creates the matrixes

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{pmatrix}$$

by multiplication of two vectors.

Practise 3: 0 Points

Check your solutions from Exercise 3 with the Matlab-function *roots*.

Practise 4: 0 Points

Plot the following shapes in one figure using the Matlab `plot` command with complex numbers as arguments.

a. Plot a blue unit circle.

b. Plot a black triangle that visualizes the addition $z_1 + z_2 = z_3$ with $z_1 = -1 - j$ and $z_2 = 0.5 + 2\text{j}$.

c. Plot a red spiral starting at the origin. The distance d to the origin grows linearly with the angle and has 3 rotations within the unit circle.

d. Plot a green spiral. Now d grows exponentially with the angle, has again 3 rotations and starts at $\frac{1}{10} + \text{j}0$.

The result should look like this: